More Mathematical Finance

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Preface

It is now ten years since the first draft of "the Concepts and Practice of Mathematical Finance" was finished. The volume of research published during that time has been immense. New areas have arisen and many questions have been resolved. Some markets such as portfolio credit derivatives have arisen, boomed and crashed. "More Mathematical Finance" is therefore a sequel, and it is intended to be a second or third book on financial mathematics. In particular, rather than recall basic theory, I will refer to "Concepts" as much as possible in order to minimize overlap and maximize the amount of new material.

This sequel is not intended to be comprehensive. The field is now far too large for such an undertaking to be practical. In any case, I am a firm believer in "write what you know." Most of the topics in the book are related to my own research in one way or another, and I hope to pass on some of the insights I have gained from using and implementing these models. To me that is the essence of the book, my objective is to give the reader my own personal perspectives on how one should view various issues. Thus whilst most of the mathematics and models here presented can be found somewhere in the literature, the perspectives I present often cannot.

Much of the book focuses on numerical methods. A pricing model is not much use unless it can be implemented and calibrated. The ability to compute Greeks is another essential. My objective is therefore to show how the mathematics can be translated into an implementable, usable model. However, this is not a recipe book. Although I present algorithms, my objective is to give the reader an understanding that makes the algorithms clear, rather than to present a piece of pseudo-code to copy out. I will, however, occasionally point to where the relevant code can be found in the QuantLib open source library. I largely avoid presenting purely numerical techniques which are well known outside finance. For example, I leave the details of how to carry out Gaussian integration to other texts. However, I do present extensive discussion of how to use Sobol numbers for quasi-Monte Carlo simulation, since this seems to be a much misunderstood topic.

I have restricted this text to mathematical finance in the sense of derivatives pricing. A more specific but rather unwieldy title might have been "how to think about some numerical techniques for pricing derivative contracts."

At some point, one must call a halt to writing, and many topics that were considered have not made it in to this book. These include Levy processes, OIS discounting, SABR, asymptotic expansion approximations, solving SDEs, numerical methods for solving PDEs, short rate models, the HJM model, commodities, power derivatives, CGMYSV, GPUs, proxy methods for Greek computation, interpolation methodologies for interest rates, local volatility, firm-value models, VAR, CES, mean-variance theory, CAPM, utility theory, APT ... The list is endless. Eventually, when I again feel that I have enough to say to justify another book, it will be time to write "Even More Mathematical Finance."

So what topics are covered? I spend four chapters on portfolio credit derivatives since it is an area that has gone from obscurity to fame to notoriety in the last few years. I look at binomial trees in depth since it is a topic which is much misunderstood: we will see that there are at least twenty different ways to place the nodes of a tree, and that each of these can be implemented in at least sixteen different ways. Monte Carlo techniques are examined in depth with chapters on the Brownian bridge, quasi-Monte Carlo, the early exercise problem and stratification.

Market models for pricing exotic interest rate derivatives have been my principal research interest for many years. This is reflected by chapters on their applicability, drift computation and approximation, correlation estimation, swap-rate market models, calibration, discounting and cross-currency market models. The chapters on discretization, factor reduction, quasi-Monte-Carlo and computing sensitivities with adjoint methods whilst written in a more general context are also directly relevant to market models.

I also include a few chapters on more philosophical questions. These include chapters on asymmetry, evaluating a perfect model, the fundamental theorem of asset pricing, convexity, mixture models and the money bleed.

Certain chapters have been included simply because I think the results and/or the mathematics are neat and I want them to share them with the reader. These include a chapter on how to differentiate the Black–Scholes formula, one on volatility derivatives and the Bachelier model. Some chapters which are both neat and very numerical are on the Fourier transform, the cos method and importance sampling with jump-diffusion models.

Few readers have the time and inclination to read a long book from start to finish and I have therefore tried to make individual chapters as independent as possible. However, there are inevitably some dependencies and I now discuss these. First, the credit chapters should be read in order. Second, the introduction to market models should be read before the other market models chapters; these are, however, largely independent of each other. The cross-currency market model chapter (26) does, of course, assume familiarity with market models. Third, the cos chapter (18) relies on the Fourier transform chapter (17). The quasi-Monte Carlo chapter (15) and the importance sampling chapter (16) both depend on the Brownian bridge chapter (14). The remaining chapters are largely stand alone and can be read in any order.

The website for this book is

`www.markjoshi.com/more`

visit there for updates, questions, new editions, typos and news. Please use the forum there to ask questions about the text and to inform me of typos.

I have included end of chapter exercises. These take a variety of forms ranging from simple computations to complicated proofs. Many of them are more computer projects than exercises since ultimately this book is about modelling. I have not included solutions, but you are encouraged to discuss the problems on the book's website.

Various versions of the manuscript have been read by a rather large number of people and I thank them all for their comments. The readers include Barbara La Scala, Will Wright, Chris Beveridge, Jiun Hong Chan, Stephen Chin, Nick Denson, Andrew Downes, Robert Tang, Chao Yang, Ferdinando Ametrano, Paulius Jakubenas, Harry Lo, Lew Burton, Agustin Lebron, Oh Kang Kwon, Alan Lewis, Nagulan Saravanamuttu, Graeme West, Dherminder Kainth and Lorenzo Liesch, as well as many others.

Mark Joshi

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